EXPLORATORY STUDIES OF LIQUID BEHAVIOR IN RANDOMLY EXCITED TANKS: LATERAL EXCITATION

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Technical Report No. 2
Contract No. NAS8-20319
Control No. DCN 1-6-57-01042(IF)
SwRI Project No. 02-1869

Prepared for

National Aeronautics and Space Administration
George C. Marshall Space Flight Center
Huntsville, Alabama

May 1967

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ABSTRACT

This report presents results obtained in an exploratory investigation of the behavior of fluid in an unbaffled circular cylindrical tank under relatively low frequency random lateral excitation. It was found that "swirl" instability may occur under random excitation and a tentative basis for predicting this instability is advanced. Attempts were made to demonstrate the validity of the linear hypothesis for the unbaffled tank under low level random excitation, but the experimental evidence developed does not appear to validate this hypothesis within the experimental range of excitation levels. Finally, brief, qualitative studies of non-stationary random excitation were carried out and are described.

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NOMENCLATURE

a	Tank radius
B_e	Effective filter bandwidth
d	Tank diameter
$\lvert F \rvert / \rho g d^3 (x_0/d)$	Modulus of (nondimensional) transfer function
$\hat{F}(x_0)$	Estimated cumulative probability
f	Frequency, cps
f_0	Center frequency (cps)
g	Acceleration of gravity
h	Depth of fluid in tank
$J_m(\ )$	Bessel function of first kind of order m
$K(t)$	Impulse response function
m,n	Indices
N	Tape speed ratio
$P(x), F(x)$	Cumulative probability
$q(A)$	Function of attenuator settings
r, ϕ, z	Cylindrical coordinate system
rms	Root mean square
$S_{F_0}(\Omega_{10})$	Parallel force spectral density at frequency Ω_{10}
$S_{F_{90}}(\Omega_{10})$	Normal force spectral density at frequency Ω_{10}
$S_X(\omega), S_Y(\omega)$	Scalar spectra of functions $X(t), Y(t)$
$S_x(f)$	Estimated spectral density at frequency f
T	Sampling time

$T(i\omega)$	Transfer function
t	Time
$X(t)$	Sample time history
$\mathfrak{X}(i\omega), \mathfrak{Y}(i\omega)$	Fourier transforms of $X(t), Y(t)$
$X(t, f_0, B_e)$	That portion of signal $X(t)$ which is passed by a narrow bandwidth filter with an effective bandwidth B_e and center frequency f_0
$Y(t)$	Sample time history
$\Delta\Omega$	Nondimensional half power bandwidth
ϵ	Phase angle
ξ_{mn}	Solutions of $\frac{d}{dr}\left[J_m(\xi_{mn}\frac{r}{a})\right]_{r=a} = 0$
ρ	Mass density
σ	Standard deviation
$\hat{\sigma}_X^2$	Variance of sample of $X(t)$
χ_{mn}	mn^{th} free surface mode
$\Psi_{XY}(i\omega)$	Cross spectrum between $X(t)$ and $Y(t)$
Ω	Nondimensional frequency
Ω_{mn}	Nondimensional Eigenfrequencies, free surface
ω	Angular frequency
ω_{mn}	Eigenfrequencies, free surface

INTRODUCTION

Since fuel and oxydizers can account for as much as 90% of the lift-off mass of a launch vehicle, the dynamic forces exerted on the vehicle by the sloshing of these liquids can be relatively large. The central problem of interest insofar as the effect of propellant motions on rocket vehicles is concerned is that of stability and control. Generally speaking, sloshing of the liquid propellants will interact with both the control system dynamics and the vehicle structural dynamics, each of which also couple with the other. As the vehicle ascends, it is subject to random aerodynamic excitation in the form of noise, wind shears, transonic buffeting, etc. These perturb the flight path and excite both structural vibration and liquid sloshing, all of which must in turn be compensated for by the control system.

In reality, then, the excitation of the liquid propellants and their containing tanks must be random in nature. Due to the nature of the thrust control and the generally light construction of rocket vehicles, both lateral and longitudinal random excitation of the fuel tanks are experienced. Though the amount of literature available on the fuel sloshing problem is very great, Ref. 1, there seems to have been no attempt to explore experimentally the behavior of fluids in tanks under any sort of random excitation. It was the objective of the present program to fill this gap by experimentally exploring the effects of random excitation of cylindrical, partially filled, rigid tanks.

The scope and purpose of work in this project may be compactly summarized by quoting from the contract:

> "Studies on liquid response to external excitations, so far, have been concentrated on deterministic processes in which the excitation of the tank is a definite function of time (primarily sinusoidal), and in which the response of the liquid is also deterministic.
>
> This study shall deal with random processes, in which the excitation of the tank and the resulting fluid response can only be described in statistical terms. Since random excitations are prevailing under low-g conditions, this study has to be considered a first step to obtain some knowledge on this particular type of liquid response which shall be the basis of further exploratory research in this field.

The statistical nature of liquid response in random processes restricts this study primarily to experimental work. The specific objectives of this study shall include both lateral and longitudinal excitation of rigid cylindrical tanks. In the case of lateral excitation, the objective is to determine the validity of the linear assumption for force response under random excitation and to explore generally the problem of lateral liquid motion induced by random excitation. In the longitudinal excitation case the objective of this project is to explore the effects of random excitation on some of the nonlinear problems of sinusoidal excitation. . . ."

The present report deals exclusively with the exploratory work carried out on random lateral excitation. The work carried out in the present program on random longitudinal excitation is dealt with in Reference 2.

PRELIMINARY DISCUSSION, LIQUID RESPONSE TO LATERAL EXCITATION

Implicit in the practical treatment of lateral sloshing for control purposes is the assumption that the forces due to fuel sloshing are mathematically linear with rigid body tank translation or rotation and their time derivatives. With this assumption, a linear differential equation may be written expressing the sloshing forces, and a transfer function relating sloshing forces to acceleration, say, may be derived. Random excitation of the fuel tanks under these circumstances, poses no insoluble problems to the design of the control system by virtue of the assumed validity of linear superposition.

The primary objective of this experimental study of fuel sloshing in response to random lateral excitation was to attempt to demonstrate the validity of the assumption that the sloshing force response under random excitation may be synthesized from the data obtained in normal fuel sloshing experiments where sinusoidal excitation is the rule. A secondary objective was a qualitative study of the randomly excited liquid free surface. Such experiments would involve giving a rigid model tank random lateral excitation, measuring this excitation and the resulting random lateral sloshing forces, and performing spectral analyses on the resulting records. (Results of a standard sinusoidal excitation experiment were also needed.)

If the relation between input $X(t)$ and output $Y(t)$ of a system is mathematically linear, the behavior of the system for arbitrary inputs may be compactly summarized in an impulse response function $K(t)$ such that:

$$Y(t) = \int_{\tau} K(\tau)\, X(t-\tau)\, d\tau \tag{1}$$

which allows calculation of the output of the system in the time domain. The transfer function ($T(i\omega)$) of the system is the Fourier transform of the impulse response function and compactly summarizes the dynamics of the system in the frequency domain. If the input and outputs of the system are absolutely integrable transients;

$$\int_{-\infty}^{\infty} \left| X(t) \right| dt < \infty$$

the transfer function is the ratio of the Fourier transforms of the output and input of the system.

$$T(i\omega) = \frac{\Upsilon(i\omega)}{\mathbf{X}(i\omega)} \tag{2}$$

If the impulse response ($K(\tau)$) for a system is known, the response to some specific random excitation may be easily calculated with Equation (1), so long as $K(t) \to 0$ for $-\infty << \tau << +\infty$. (In effect the integrand is then non-zero over a limited range.) There is no reason for a random function to satisfy the conditions of integrability previously cited and thus Equation (2) is not directly applicable. However, the methods of generalized harmonic analysis allow an analogoud relationship:

If $\Psi_{XY}(i\omega)$ is the cross spectrum between input $X(t)$ and output $Y(t)$, (both random functions)

$S_X(\omega)$ is the scalar spectrum of input $X(t)$

$S_Y(\omega)$ is the scalar spectrum of $Y(t)$

Then
$$\hat{T}(i\omega) = \frac{\Psi_{XY}(i\omega)}{S_X(\omega)} \tag{3}$$

and
$$\left|\hat{T}(i\omega)\right|^2 = S_Y(\omega)/S_X(\omega) \tag{4}$$

Since the true spectra and cross spectra of a random process involve calculations of limiting values as time increases without bound, we can only estimate spectra and cross-spectra from a finite sample with some statistical measure of confidence. Consequently, Equations (3) and (4) express <u>estimated</u> properties of the system.

The conventional method of determining transfer functions experimentally is to excite the system with pure sinusoidal excitation of various frequencies, and note the output amplitude and phase relative to the input.

If the fuel sloshing force response is truly linear, all three cited methods of obtaining the transfer function (Equations (2), (3) and (4) and the previous paragraph) should yield the same result, within the practical experimental limitations imposed on each method.

Consequently, a demonstration of linearity (or the lack of) could be made by exciting a tank laterally with both sinusoidal and random displacement or acceleration signals, calculating the transfer function in each case and comparing the results. (The estimates of Equations (3) and (4) can be made to an accuracy comparable with conventional procedures.)

It may be noted that the force response of laterally excited tanks cannot be expected to be linear if excitation magnitude is increased indefinitely. One of the important advantages in embarking on random tank excitation studies is that such studies may ultimately indicate how far the linear model is to be trusted in terms compatible with the type of excitation time histories which can be measured in the vehicle, rather than in terms of discrete sinusoidal excitation which persists indefinitely.

In the preceding paragraphs, the fluid in the tank is essentially treated as a linear "black box". As may be seen from Chapters 2-4 of Ref. 1, this is by no means always true and in the present program considerations of what types of nonlinear behavior might be encountered influenced the detailed experimental approach.

Summarizing the common types of nonlinear fluid behavior (after Ref. 1, Chapter 3), nonlinear effects in lateral sloshing might be described in terms of four classes:

1. Nonlinearities which arise primarily as a consequence of the geometry of the container and are apparent even at very small excitation levels. Examples of this class are compartmented (sector) cylindrical tanks, spherical tanks, spheroidal, etc. The circular cylindrical tank does not appear to have significant nonlinearities of this type. Since the circular cylindrical tank is one of the most common geometries used in practice this type was selected for the present experiments.

2. Nonlinearities which arise primarily as a consequence of large amplitude excitation and response. This type will occur in any tank if the excitation level is made sufficiently high. Consideration of such response indicated for the present program that the experimental apparatus should be capable of as large an excitation level as possible.

3. Nonlinearities which involve essentially different forms of liquid behavior produced by coupling or instabilities of various of the lateral sloshing modes. The most important of these seems to be "rotary sloshing" or "swirl motion". This is a type of instability occurring very close to the lowest liquid resonant frequency. Quoting the qualitative description of Ref. 1, Chapter 3, which summarizes observations from <u>sinusoidal</u> excitation experiments: "The essential features of this complex liquid motion can be

described qualitatively as an apparent "rotation" of the liquid about the vertical axis of symmetry of the tank, superimposed on the normal sloshing motion. The motion is even more complicated as a type of "beating" also seems to exist; the first antisymmetric liquid-sloshing mode first begins to transform itself into a rotational motion increasing in angular velocity in, say, the counterclockwise direction, which reaches a maximum and then decreases essentially to zero and then reverses and increases in angular velocity in the clockwise direction, and so on alternately. The frequency of rotation is less than that of the surface wave motion and therefore the liquid appears to undergo a vertical up-and-down motion as it rotates about the tank axis; the rotational frequency about this up-and-down axis is about the same as that of the wave motion. The liquid free surface, at least at relatively low excitation amplitudes, is essentially plane, and it is the apparent rotation of this inclined plane about a vertical axis that we are attempting to describe. This phenomenon almost invariably occurs in laboratory tests at frequencies in the immediate neighborhood of one of the resonances of the normal sloshing modes, as mentioned above, and occurs whether the liquid has any initial gross rotation or not; the rotational mode can, however, be initiated at any excitation frequency by introducing some disturbance which provides a substantial initial rotation to the fluid."

In practice, both experimental and actual, it has been found that baffles internal to the tank tend to suppress this instability, especially if in the form of vertical plates. With respect to the present experiments, it was thus necessary to a) find out if "swirl" was possible at all under random excitation, b) consider whether or not to incorporate swirl suppression devices in the test tank.

4. The last type of possible nonlinearity in the system is that produced by the introduction of large damping in the system, as by baffles or other mechanical devices. Unfortunately, the common ring-baffle is at least a slightly nonlinear device. In the present program the possible incorporation of baffles in the test tank was considered in conjunction with the "swirl" problem above.

EXPERIMENTAL APPROACH

In random excitation the describing parameters are necessarily slightly more complex than for sinusoidal excitation where a discrete frequency and an amplitude (of displacement or acceleration) suffice. For any random excitation at least 3 describing parameters and a specification of a probability distribution are required. In a practical sense the underlying probability distribution of experimentally produced random excitation very nearly has to be Gaussian (normal) and "stationary". The "normal" specification is because of the nature of available equipment, the fact that the literature on the normal distribution is the most extensive, and the fact that the result of a linear operation on a random Gaussian process is another random Gaussian process. The normal distribution is completely described by the mean and variance of the process (the mean squared deviation from the mean). In the present case it is more convenient to specify variance, or root-mean-square amplitude (abbreviated rms hereafter). The speficication "stationary" means loosely that the statistical properties (variance, moments of distribution, etc.) do not change with time.

The rms amplitude of the excitation is not sufficient in itself to define the excitation for present purposes. The frequency band with which most of the energy (or mean square) is to be associated is also required and this must be defined by at least two parameters. Thus, experimentally it would be necessary to deal with three parameters rather than two. In addition, in the random excitation case, the excitation would contain energy over a continuous range of frequency.

The question of the possibility of "swirl" instability under random excitation was of importance at the outset since some excitation energy would always be present near the first anti-symmetric mode. Thus large swirl suppression baffles might be a necessary complication to the tank geometry. There was no way to know in advance how important to the experiments swirl might be, and the final decision on tank baffling was deferred pending preliminary experiments with an unbaffled cylindrical tank.

It happened that the mechanical equipment and tank used and described in Reference 3 was available. Though the associated electronic excitation equipment was not immediately suitable for the random excitation case, it was decided that the most economical course to follow in the present investigation would be to modify the equipment of Ref. 3, as necessary. The 7.7 inch ID circular cylindrical tank (19.6 cm) was accordingly stripped of old baffles and, when it became possible to assemble the lateral random excitation system (to be later described), a qualitative study was undertaken of the "swirl" tendencies in an unbaffled tank subject to random lateral excitation.

The frequency bandwidth of the random excitation was varied and the nature of the free surface response was noted. Little or none of the spectacular rotational instability associated with sinusoidal excitation near the first anti-symmetric resonance was observed in any case. As the frequency bandwidth was reduced to a width covering the range from DC to about that of the second anti-symmetric mode, more and more obvious first anti-symmetric mode response was observed. This response built up and died out as was to be expected but displayed only an occasional tendency to rotate. Sinusoidal excitation experiments were carried out to be sure that the lack of rotation was not associated with the experimental setup, but substantial rotation was observed with a pure sinusoidal excitation energy input an order of magnitude less than the energy in the highest level of random excitation. These results implied that in order to produce pronounced rotational instability under random excitation, the excitation input to the system must at least have a very high relative energy level near first anti-symmetric resonance.

Some brief additional experiments on transient excitation were also carried out. If the gain of the stationary random excitation signal is varied with time, the excitation becomes one of a class of nonstationary random processes. In the experiments the stationary random signal which produced the most qualitative tendencies for rotation was used. The gain of this signal was carried from zero to nominal maximum and back to zero in a period of time sufficient to produce a lateral excitation transient similar to those sometimes observed in the flight of a booster vehicle, Ref. 1. This procedure was repeated several times to excite the unbaffled tank. In each case the tank fluid was initially quiescent. The random transient excited many modes initially but after the excitation ceased, the fluid decayed in the first anti-symmetric mode with no obvious rotational tendencies. Quite similar behavior was observed when the tank was given a displacement "step" excitation (a velocity impulse).

No measurements were made, and thus no reasonably precise idea of the distribution of energy in the excitation could be obtained in these preliminary experiments. However, they indicated that, within the spirit of an exploratory program, more might be learned if the first test tank was unbaffled and the fluid left free to rotate. In a preliminary way it seemed possible that rotation would not occur in an unbaffled tank with "broadband" random excitation. Under these circumstances, sufficient checks on the linearity of the system might be made. In addition, an enlargement of the program to include a preliminary study of the conditions required to initiate rotational instability appeared to be worthwhile.

The final experimental program was thus, in general, to give an unbaffled circular cylindrical tank random displacement excitation having a wide range of frequency distributions, and to record the resulting forces imposed on the tank by the fluid. Displacements were selected as the "input" as a matter of experimental convenience. The resulting forces are of more direct bearing on practical sloshing problems, and were also experimentally convenient since apparatus previously constructed under NASA contract was available.

EXPERIMENTAL APPARATUS

1. Mechanical Apparatus, Tank

The mechanical apparatus used and described in Ref. 3 was available for the present exploratory program. It was felt that the most economy would result for the present program if this apparatus was modified where necessary. One of the test tanks of Ref. 3 was also available and this identical tank was used in the present experiments. This apparatus was considered slightly deficient in displacement excitation range for present purposes (about $\pm$ 0.017 tank diameters). This deficiency was far outweighed by economic considerations and by the availability, in Ref. 3, of the results of sinusoidal excitation experiments performed on the same cylindrical tank to be used in the present program.

An overall view of the setup used in Ref. 3 is shown in Figure 1 of this report. The mechanical apparatus consists of a base supporting all parts of the system, an electro-dynamic vibration exciter, a carriage which may be thereby excited in translation, and which, in turn, supports the test tank through a force balance system. It was necessary to stiffen the base to minimize vibration interaction with the shaker control system to be later described, and it was felt prudent to replace the four teflon carriage slide bearings with three linear ball bearings for the present application. The only other mechanical addition to the existing apparatus was the addition of a linear motion carbon film potentiometer to measure relative displacement between carriage and base.

The tank itself, Figure 1 and 2, is a 7.70 inch ID (19.6 cm) Lucite plastic cylinder with a flat bottom. Tap water (at about 70^{o}F) was the liquid used, and the tank was filled to a depth of one diameter for all experiments herein. The top of the tank was covered with plastic food wrapping material to prevent loss of water.

The tank was attached to the carriage through the four special force balances described in Ref. 3 (one visible in Figure 2). These balances are provided with strain sensing elements both parallel and normal to the direction of excitation. A cantilever spring-mass "accelerometer" whose sensitivity may be adjusted by adjusting mass, is also provided. Suitable connection of all the semi-conductor strain gages in the force balances and accelerometer is possible such that output signals may be obtained which are proportional to the net force exerted on the tank by the fluid both parallel and normal to the direction of excitation. Near-cancellation of the inertia signal of the empty tank is achieved by varying the mass on the "accelerometers". Deadweight calibrations of the force measuring system were provided for with pulleys and strings, Fig. 2.

2. Recording Apparatus

The electronic equipment involved in the present experiments conveniently falls into three categories: 1) Shaker control and power, 2) Excitation and, 3) Recording apparatus. Figure 3 shows most of the apparatus used. Shaker and Excitation systems will be discussed in succeeding sections.

The basic recorder in the present experiments was an Ampex FR 1800L magnetic tape recorder. For any given excitation condition, two force "outputs" and a displacement "input" were recorded simultaneously for subsequent analysis. Two Tektronix Type "Q" Transducer and Strain Gage Units were employed to excite, amplify and demodulate the two force bridge signals. The outputs of these amplifiers were low pass filtered by SKL Variable active filters to eliminate high frequency noise corresponding to force balance resonances, etc. Electronic calibrating signals are available in the "Q" units and these were used to put "step" calibrations on the magnetic tape. The amount of force corresponding to these calibration signals was established through static deadweight calibrations performed at the start and checked at the end of each test day. The carbon film potentiometer displacement transducer was used for recording "input" and as an input to the shaker control system. A buffer amplifier in the shaker control system provided a displacement signal output and this signal was further amplified or attenuated as required by means of a Tektronix Type "O" Operational Amplifier Unit. A stable voltage calibration signal was also provided for the displacement signal and was recorded on the magnetic tape before and after each "run". The physical displacement corresponding to the calibration signal was established through a static calibration of the carriage which was performed after the static force calibrations.

3. Shaker System

The excitation signal (to be described) is basically a fluctuating voltage at the "input" to the shaker system. In the experiments it is most convenient to change the nature and/or the frequency content of this voltage. In practice, if the investigator wishes to have some notion during the experiment of the frequency distribution and level of acceleration he is actually imposing on the specimen it is necessary to "equalize" the shaker system so that a given sinusoidal voltage input amplitude results in an essentially constant displacement amplitude over the frequency range of interest. The 50-lb force electrodynamic shaker system utilized in Ref. 3 was not really intended for use below 5 cps and consequently had a sharply varying frequency response below this frequency. It was, nevertheless, adequate for the sinusoidal excitation experiments of Ref. 3 because "equalizing" (in the sense used above) for sinusoidal experiments is greatly facilitated by the fact that only one frequency at a time need be handled. In random excitation experiments the system must be equalized for all significant frequencies all the time. The main difficulty with the existing 50-lb shaker system was that the shaker was AC coupled to the power amplifier. This was the identical problem encountered in the previous portion of the present program (Ref. 2) and the solution was identical; that is, a small direct coupled solid state power amplifier used in the previous phase of the program was used in place of the 50-lb shaker's normal power amplifier. This substitution extended the range of shaker operation to DC but, of course, did not complete the equalization. By utilizing the solid state power amplifier, a displacement signal from the previously mentioned carbon film potentiometer, and a velocity signal from a transducer built into the shaker, it was possible to design and construct a closed loop equalization system, or control servo. The final shaker system, loaded to simulate the mass of the fluid in the tank, had a flat (within 1%) frequency response from DC to 5 times the frequency of the first anti-symmetric mode with a relatively gentle roll-off thereafter. The relatively tight "loop" in the system made it quite versatile. For example, a quite creditable "step" function could be achieved (rise times were very small relative to time scales appropropriate for sloshing) as well as ramps, triangular waves, etc. In addition to good frequency response characteristics, there was sufficient power to drive the system into the mechanical stops ($\pm$ 0.017 tank diameters) for signals within the "flat" frequency range. Consequently, a clipping amplifier was inserted between shaker and excitation systems to limit the excitation voltage to a range corresponding to the full mechanical deflection range. The input to this amplifier became the effective input to the shaker system.

4. <u>Excitation System</u>

The central random excitation requirement is a source of Gaussian noise with uniform distribution of energy over the frequency range of interest. Since the frequencies involved in sloshing problems are below the audio range, a special low frequency noise source is convenient and in the present case an Elgenco Corp. Model 311A source provided a reliable Gaussian noise with frequency content from DC to 40 cps.

In the test program, the rms tank displacement level and frequency distribution were to be varied. This was done by interposing between the noise source and the input to the shaker system, various combinations of frequency band shaping filters (to be described) and a buffer amplifier with step input attenuator. The buffer amplifier allowed adjustment of gain so that tank displacement amplitudes up to just under limiting could be achieved for each combination of shaping filters, the step attenuator was used to vary the excitation level.

During the experiments three distinct methods of shaping noise were employed. The first, for "broadband" noise, was to filter the noise through two cascaded SKL variable cutoff active filters. One of these was set to control the low frequency cutoff, the other for the high frequency cutoff. The second method, for narrow band noise, involved the use of a Spectral Dynamics Corp. Tracking Filter with a (fixed) 2 cps bandwidth filter. With this equipment a 2-cycle wide band of noise at any desired center frequency could be picked out of the output of the noise source.

Since a 2-cycle bandwidth is actually "broad" relative to the separation of modal frequencies in the test tank, a method was sought to produce much narrower band random excitation signals. It was found that (with some care) it was possible to play back a signal previously recorded on one track of the Magnetic Tape Recorder while simultaneously recording data on other channels. This capability allowed the production of 1/2 and 1/8 cycle band noise by time scale division of 2-cycle band random noise samples. By recording 2-cycle band noise at various tape speeds, excitation signals of various smaller bandwidths were produced upon playback at the lowest (recording) tape speed.

ANALYSIS EQUIPMENT

Analyses of random data commonly involve the estimation of the spectral distribution of "energy" (mean square deviation) of the process from a sample, the estimation of total mean square and, less commonly, estimation of the probability distribution function. The availability of the recorded samples of acceleration and free surface elevation on magnetic tape made analog analyses feasible. The lowest frequencies present in the data were such that playback of the tape 4 or more times faster than the recording speed resulted in apparent frequencies which fit comfortably within the capabilities of available analysis equipment. However, no loop recorder was available and thus any analog analyses attempted would have to be compatible with "shuttling" of tape back and forth, that is the rapid continuous frequency scanning methods in common use would have to be modified.

1. Spectral and Mean Square Estimates

Given a sample voltage time history $X(t)$ from a stationary random signal, the power spectral density function $S_X(f)$ for the signal may be estimated as in Eq. 5 (Ref. 4, for example),

$$\hat{S}_X(f) = \frac{1}{B_e T} \int_0^T X^2(t, f_o, B_e)\,dt \tag{5}$$

where:

$\hat{S}_X(f_o)$ = Estimated Spectral Density at Frequency f_o

$X(t, f_o, B_e)$ = That portion of the signal $X(t)$ which is passed by a narrow bandwidth filter with an effective bandwidth B_e cps and center frequency f_o cps.

T = Sampling Time

In words, the estimates are made by the following operations:

1. Frequency filtering of the signal by a narrow bandpass filter having an effective bandwidth of B_e cps and a center frequency of f_o cps.

2. Squaring of the instantaneous value of the filtered signal.

3. Averaging of the squared instantaneous value over the sampling time T.

4. Division of this mean square by B_e.

In the present case equipment was available to approximate the integral of the square by both RC filtering of the rectified, filtered signal, and by performing the square and an integration. However, nonlinear response, and thus non-Gaussian, signals were expected and the true mean square measurement was to be preferred.

An annotated functional diagram of the spectrum analysis system used is shown in Figure 4 and the physical setup in Figure 5. Referring to Figure 4, the first operation is a compression of time scale. The narrow band filtering was carried out by the same tracking filter used to generate some of the excitation signals. The center frequencies were set manually. The filtered signal was then attenuated as required to suit the square-law output of a true rms voltmeter. The output of the square law device was then frequency modulated and the number of cycles in this FM signal were counted over a controlled time interval. As noted in Figure 4, this count is proportional to spectral density. In order to calibrate this count the random signal was replaced by a known sinusoidal voltage and the system operated as for the actual spectrum analysis.

In operation, tape play back speed, filter bandwidth, and sampling time were selected, set up and recorded, step calibration signals on the tape before and after the sample were measured and the system constant for converting counter display to (volts)^2 was established. The operator started tape playback, selected a center frequency (f), and Ballantine attenuator (A) to yield a reasonable counter display, started the count of the sample, recorded the total count during the sampling time, and then adjusted the center frequency to the next value, and so on. When it was judged that sufficient point coverage of the frequency range had been achieved the operator moved on to another sample on the tape. Ultimately, a number representing the physical calibration of the data on the tape was added to the recorded data, all numbers key punched and in the CDC 160A digital computer operated by the SwRI Computations Laboratory, the analysis was completed by a) applying calibration constants compensated for attenuator settings to the counter outputs to convert to (engineering units)^2, b) this result was divided by bandwidth, B_e compensated for tape playback speed to result in dimensional spectral density, c) these spectral densities and the associated frequencies were non-dimensionalized, d) tabulated and e) rough plotted.

The variance of the sample, $\hat{\sigma}^2$, was estimated with the same equipment by by-passing the tracking filter, Figure 4. In this case the counter output is proportional to $\hat{\sigma}_X^2$ where:

$$\hat{\sigma}_X^2 = \frac{1}{T}\int_0^T X^2(t)\,dt \tag{6}$$

Much the same procedure in calibrating the system for spectral density estimation was followed for variance estimation.

The selection of bandwidths, sampling times, tape playback speeds, etc., for the analysis of each taped data sample was made from statistical considerations and these will be discussed in a subsequent section.

2. <u>Probability Distribution Estimates</u>

Because an analog system for cumulative probability estimates $P(X)$ is easier to realize than one for probability <u>density</u> estimates, $\left(\frac{d}{dX}P(X)\right)$, this approach was chosen. If δt_i is the time spent by $X(t)$ above the level X_o during the i[th] excursion above X_o, the cumulative probability may be written (Ref. 4):

$$P\left(X(t) > X_o\right) = \lim_{T \to \infty} \left[\frac{\sum_i \delta t_i}{T}\right] \tag{7}$$

and thus an estimate (Ref. 4) for the cumulative probability from a record of $X(t)$, T seconds long is:

$$\hat{F}(X_o) = \hat{P}\left(X(t) > X_o\right) = \frac{1}{T}\sum_i \delta t_i \tag{8}$$

The method by which this estimate was realized is outlined in Figure 6. The sample signal from the magnetic tape was compared with a manually adjusted voltage reference (E_o) in a voltage comparator. The output of the voltage comparator actuated a gating amplifier which turned on and off a high frequency sinusoidal signal according to whether the sample signal was above or below the reference. This gated signal was counted over the time necessary for an integral power of ten cycles of the high frequency sinusoidal signal. Thus the need to perform any arithmetic on the accumulated count was eliminated (apart from the decimal point) and the count was written down as the estimate, $\hat{F}(E_o)$. The process was repeated for as many reference levels as needed to define $\hat{F}(E_o)$ reasonably within the $\pm$ 3-1/2 σ range on normal probability paper.

Because the period of the gated high frequency signal is finite and because the voltage comparator and gate do not switch instantaneously, there are errors in the process. These were minimized by maintaining the frequency f_c (Figure 6) at 1000 cps, which was between 50 and 250 times the highest significant frequency in any of the records. The system was tested with sinusoidal inputs to establish the required magnitude of f_c , among other things, and it is felt from the good correlation and repeatability achieved with sine waves that the inherent errors in the estimate of $\sum_i \delta t_i / T$ will be far less than the sampling errors involved in relating $\hat{F}(E_o)$ with $P(E_o)$.

DIMENSIONAL CONSIDERATIONS

In the experiments, tank displacement, and forces parallel to and normal to the tank displacement were recorded as functions of time. In subsequent analyses of data, time as a parameter was replaced by frequency. While no representations are intended that the data to be presented can be extrapolated to any size tank, a non-dimensional presentation ordinarily facilitates comparison with other experiments and with analytical results.

A convenient form for frequency nondimensionalization is

$$\Omega = \omega\sqrt{\frac{d}{g}} \tag{9}$$

where: ω = angular frequency
the excitation, tank displacement, was divided by the tank diameter (d) to be consistent with past results (Ref. 1). A convenient form for force nondimensionalization is:

$$\text{Nondimensional Force} = \frac{(\text{Forces})}{\rho g d^3} \tag{10}$$

where

ρ = mass density of fluid.

These nondimensionalizations were carried through the spectral analyses to result in excitation spectra having dimensions, $(\text{diameter})^2/\delta\Omega$ and force spectra having dimensions of (units of $\rho g d^3)/\delta\Omega$

For convenience, the first twenty eigenfrequencies of the free surface are tabulated in Table I in terms of the nondimensionalization of Eq. 9. The subscript $n=0$ herein refers to the lowest frequency of the type of mode, m ; that is, the mode shapes are assumed of the form:

$$X_{mn} \propto \cos(m\phi+\epsilon)\, J_m(\xi_{mn}\tfrac{r}{a}) \tag{11}$$

TABLE I

The First Twenty Eigenfrequencies for a Circular Cylindrical Tank (h/d = 1)

Frequency Order	m	n	Ω_{mn}	Remarks:
1	1	0	1.92 ←	First Anti-Symmetric
2	2	0	2.47	
3	0	0	2.76 ←	First Axi-Symmetric
4	3	0	2.90	
5	4	0	3.26	
6	1	1	3.265 ←	Second Anti-Symmetric
7	5	0	3.58	
8	2	1	3.66	
9	0	1	3.75	
10	6	0	3.87	
11	3	1	4.00	
12	1	2	4.13 ←	Third Anti-Symmetric
13	7	0	4.14	
14	4	1	4.31	
15	8	0	4.39	
16	0	2	4.51	
17	5	1	4.58	
18	3	2	4.76	
19	1	3	4.84 ←	Fourth Anti-Symmetric
20	6	1	4.86	

where:

r = radial coordinate of a point on the free surface

ϕ = angular coordinate of a point on the free surface

m = 0, 1, 2 ....

n = an index, 0, 1, 2...

ϵ = a phase angle

$J_m(\ )$ = Bessel function of first kind of order m .

ξ_{mn} = solutions of: $\frac{d}{dr}\left[J_m(\xi_{mn}\frac{r}{a})\right]_{r=a} = 0$

a = tank radius

Eigenfrequencies are defined by:

$$\Omega_{mn}^2 = \frac{\omega_{mn}^2}{g}\, d = 2\,\xi_{mn} \tan h\, (\xi_{mn}\frac{2h}{d})$$

$$\approx 2\,\xi_{mn}\,,\ \frac{h}{d} = 1 \qquad (12)$$

where h = depth of fluid in tank, and the ξ_{mn} are tabulated in Ref. 1.

EXPERIMENTAL PROGRAM

Since some sinusoidal results for the test tank were available from Ref. 3, no sinusoidal excitation experiments were carried out. The excitation conditions in the test program are summarized in Table II, and typical excitation spectra are shown in Figure 7. The program was oriented toward a study of swirl instability as well as obtaining some checks on linearity. As may be seen in the table, the excitation bandwidth was varied from an extremely narrow case having frequency content only in way of the first anti-symmetric mode, to a broadband case covering the frequencies of about the first 100 free-surface modes. In each case the excitation level was varied from maximum obtainable within mechanical stops on the equipment to about 1/10 this level, rms (about 1/100 energy). All the spectra of Figure 7 are for the maximum excitation levels. Spectra corresponding to 1/100 energy for each case were observed to have the same shape but displaced downward two orders of magnitude in spectral density.

The qualitative study of non-stationary random excitation was carried out by varying the gain of the stationary "band-passed" excitation signal from zero to maximum and back to zero by hand, in a period of time equivalent to between 10 and 20 periods of the first anti-symmetric mode.

After data taking proper, experiments were carried out to a) determine the frequency response of the filters used in conjunction with the two force records and b) obtain sufficient data to determine the force "tares". (Despite the provisions made in the apparatus to automatically subtract inertial tares from the total force signals some "residual" force is always present due to imperfections in mechanical and electronic equipment. The contribution of the residual tare forces to the total force spectra was later found to be negligible.) Finally, motion pictures were obtained of selected test conditions. The Appendix to this report contains notes on these motion pictures.

TABLE II

EXPERIMENTAL PROGRAM-RANDOM EXCITATION

Type of Excitation	Half-Power Bandwidth $\Delta\Omega$	Center Frequency	Tank Displacement Levels (Diameters)	Number of Samples	Method of Data Reduction
Narrow Band-1	0.11	Ω_{10}	6.35×10^{-3} rms and 8.16×10^{-4} rms	2	Analog
Narrow Band-2	0.45	Ω_{10}	5.85×10^{-3} rms and 7.52×10^{-4} rms	2	Analog
Band Passed	1.77	Ω_{10}	5.08×10^{-3} rms and 6.97×10^{-4} rms	2	Analog
Broad Band	Significant Energy Over Frequency Range of First 100 Free Surface Modes		4.49×10^{-3} rms 1.66×10^{-3} rms 5.3×10^{-4} rms	3	Digital + Analog
Non-Stationary Random Excitation, "Band Passed" Stationary Spectrum	Produced by Variation of Gain of "Band Passed" Excitation Signal		Underlying Stationary Signal Approx. 5×10^{-3} rms	3	None (Qualitative Study)

DATA REDUCTION

1. Analog Analyses

The mean square measurements were carried out with a constant real time sampling interval, such that the minimum product: (sample time x effective energy bandwidth in the sample) was approximately 30. This implies a standard statistical error on root-mean-square estimates (Ref. 4) of approximately $\pm$ 10%, or in terms of a confidence statement for the root-mean-square values to be quoted: it may be said with about 90% confidence that the true rms value of the process is at worst within $\pm$ 15% of the estimate.

The choise of all analyzer bandwidths and integrating times for each spectral estimate was such as to make the product: analyzer bandwidth x sampling time equal to 60. This is equivalent to a standard error of about 13%, or, if the spectrum is "resolved" adequately, it may be said with about 90% confidence that the true spectral density is within $\pm$ 20% of the quoted estimate.

It is unfortunate that in order to adequately "resolve" a spectrum, it must be known in advance. In the present analyses the tank displacement spectra could be fairly closely estimated in advance and suitable resolution determined by choosing an analyzer bandwidth 1/4 or less of the half power bandwidth of the spectral peak. Force spectra were similarly resolved, though in most cases, inadequately. In view of the exploratory nature of the program objectives, the use to be made of the spectra did not justify a second, more refined analysis. Of the results to be presented it is felt that the tank displacement spectra are reasonably well resolved, while, where relatively sharp spectral density peaks occurred for forces, the analysis procedure probably distorted reality by lowering the maximum and broadening the bandwidth of the true spectral peak.

Cumulative probability distributions were estimated with the previously described equipment for selected samples in the test program. Since the basic noise source was Gaussian, the tank displacement samples should have been Gaussian. This was verified in one or two cases. Most of the samples selected for probability analyses were of fluid induced forces. The forces at 90° to the direction of excitation were expected to be non-linear and non-Gaussian. Each estimated point was plotted on normal probability paper as it was obtained. These determinations of probability are effectively converted to the nominally equivalent standardized normal variate when plotted on normal probability paper. Since the statistical properties of the time intervals which make up the estimate $\hat{F}(E_0)$, Eq. 8, are not known, confidence statements about the estimates cannot be made. On an intuitive basis as in Ref. 2, the sampling time, T , for all estimates was made the same and selected so that the length of record analyzed was equivalent to 800 times the period of the lowest period phenomena involved (the period of the first anti-symmetric mode). By analogy with digital sampling techniques, if the T seconds of recorded $x(t)$ had been sampled at an interval appropriate for digital analyses of the probability distributions, in no case should the number of degrees of freedom (the number of statistically independent samples) have been less than 100. Thus, a respectable statistical sample, at least, is indicated.

The portions of the program where analog data reduction methods were employed are noted in Table II.

2. Digital Analyses

The three "broadband" excitation cases of Table II were reduced by digital techniques since it was desired to estimate cross-spectra, and this was not possible by analog methods with equipment in hand.

Semi-automatic chart reading equipment was available, and samples of the recorded data on magnetic tape were "played out" on a direct writing oscillograph. The resulting oscillograph traces of tank displacement and two fluid forces were sampled at equi-spaced intervals of time short enough to resolve the highest frequency present so that digital spectrum and cross spectrum analyses could be carried out with the resulting time series. The programs used in carrying out the analyses are those of Ref. 5. The actual computations were carried out by the Computations Laboratory of Southwest Research Institute on a CDC 3600 general purpose digital computer.

As in analog analysis, it is highly beneficial to know the answer before starting the analysis. In the present case it was possible to estimate the excitation spectram form fairly closely, but the form of the force spectra was unknown. On the basis of economics in an exploratory program and an (erroneous) guess on the width of the spectral force peak at first anti-symmetric mode resonance, measurement of 1000 points per sample was felt adequate.

After the analog analysis had been accomplished it was seen that the first anti-symmetric mode spectral peak for force was narrower than anticipated and the digital analysis had to be setup to produce estimates at a frequency interval of 0.213. This resulted in an analysis with 20 "degrees of freedom", lower than had been hoped for. Confidence bounds (90%) on derived amplitude and phase results for these analyses would be $\pm 18\%$ on amplitude response and $\pm$ 10^{o} on phase if a "coherency" (Ref. 4) of around 0.90 was attained.

A trial analysis was made and it was seen that the computed coherencies near first anti-symmetric resonance were extremely low, and that the spectral peak was as narrow as in the analog analysis results. Low coherencies indicate nonlinear response if resolution is adequate, and it was obvious that resolution on the frequency scale was inadequate. In digital analysis increase in resolution is accomplished very simply but at the expense of statistical confidence. Unless two or three times the number of points already read from the records had been obtained for use with a higher resolution analysis, such an analysis would have been no improvement. In order to avoid the measurement of more points from the record, a special "pre-whitening" filter was devised (Ref. 6). This filter was essentially a numerical "notch" filter which operates on the original time series and sharply attenuates the frequency components near first anti-symmetric mode resonance. This operation, (with luck) makes the estimated spectrum from the "pre-whitened" record flat and thus amenable to a relatively coarse resolution on the frequency scale. After the spectrum analysis the spectra are "re-colored" by multiplying by the square of the reciprocal modulus of the pre-whitening filter transfer function. In the present case the prewhitening filter was designed to introduce no phase distortion. All is not gained without payment, however. The pre-whitening operation reduced the number of data points to 884 from 1000 and thus the "degrees of freedom" to 17 instead of 20. Confidence bounds (90%) on amplitude and phase of transfer functions derived from the cross spectral analysis would then be $\pm$ 20% on amplitude response and $\pm$ 12^{o} on phase if a coherency of at least 0.90 was attained. Since coherencies greater than 0.9 were quite prevalent in the trial analysis for force in direction of excitation at frequencies other than first anti-symmetric mode resonance, the pre-whitening method was chosen for the final analyses.

RESULTS AND DISCUSSION

1. Rotational Instability Under Random Excitation

The primary purpose in the inclusion of narrow band excitations in the program (Table II) was to study swirl instability under random excitation. Qualitative observations indicated that under "Narrow-Band-1" excitation (half power bandwidth = 0.11) the free surface behaved similarly to that expected for sinusoidal excitation for the higher excitation level. In particular, violent rotation or swirl was observed to build up, decay, and change direction in a manner analogous to sinusoidal excitation experience. Under this same very narrow bandwidth, excitation at 1/100 maximum energy level, little rotation could be observed visually. The "Narrow Band-1" excitation is nearly a randomly modulated sine wave and this qualitative result is perhaps not surprising.

As bandwidth was increased to Narrow Band-2" (Ω = 0.45) the same qualitative behavior of the free surface was observed though rotation was perceptably less violent. In the "Bandpassed" excitation case (half power band 1.77) very little rotation could be observed visually at either excitation level. No significant rotation was observed visually in the "Broadband" excitation case.

The visual observations cited above were, of course, much less sensitive than the force measurements. When all the analyses of force spectra were completed it was noted that the shapes of the spectra near first anti-symmetric mode resonance were essentially the same regardless of excitation bandwidth.

The force spectra resulting from "Narrow Band-2" excitation are typical and are shown in Figure 8. For the higher excitation level, rotation was observed visually and it may be seen that the spectrum of force normal to the direction of excitation is almost identical with the spectrum of force parallel to the excitation. When excitation energy level was reduced by a factor of 80 to 100 it may be seen from Figure 8 that the spectrum for force normal to direction of excitation is about 1/100 the spectrum of force parallel to the excitation.

The similarity in shape of the force spectra near first anti-symmetric mode resonance, indicated that correlations might be made on the basis of spectral densities at first anti-symmetric mode frequency (essentially the peaks in the spectra). Figure 9 is the result of such a correlation. The abcissa of Figure 9 is the tank displacement spectral density at the first anti-symmetric mode. The ordinate is a ratio of the peak spectral density of the force normal to direction of excitation, to the peak spectral density of force parallel to excitation. When this ratio is 1.0, violent rotation exists, when it is very small, little significant rotation was experienced. The lines in Figure 9 are more for identification of points than to indicate trends. (It should be noted that the omission of the point corresponding to the low level "Narrow Band-1" case was because maladjustment of attenuators during the experiment produced a normal force record almost impossible to analyze.)

Within the limitations of a small amount of data it seems that the important excitation parameter defining the onset of significant rotation is the excitation spectral density at the first anti-symmetric mode frequency. For the particular test tank under investigation, it appears that below a tank displacement spectral density of 10^{-6} $(\text{diameter})^2/\delta\Omega$ no significant rotation is to be expected. Above this figure quite violent rotation may be experienced.

The above figure for a random "stability" boundary seems to be of the correct order of magnitude relative to sinusoidal experience. The swirl stability boundary given in Ref. 3 is shown in Figure 10 of this report (in non-dimensional units consistent with present notation). Within the range of rms displacement possible in the experimental apparatus, "swirl" instability occurs over a frequency bandwidth of about 0.25. If rms sinusoidal excitation is below 0.46×10^{-3} diameters no swirl results. The total mean square displacement in an 0.25 wide frequency band for a random tank displacement excitation with spectral density of 10^{-6} is (0.25×10^{-6}), or an rms displacement of (0.5×10^{-3}). This last figure is in correspondence with the lowest point on the stability diagram, Figure 10. The good numerical correlation should not be taken seriously since the critical bandwidth to be associated with the spectral density is an hypothesis formed for the occasion. However, the magnitude of the spectral density for the onset of swirl does seem consistent with sinusoidal experience.

2. Probability Distributions

As mentioned in the section on data reduction, the probability distributions of tank displacements were checked in one or two cases and the results indicated normality. Six of the nine records of force parallel to the direction of excitation were checked. Though some slight suspicion of deviation of this response from normality existed for records where violent rotation was observed, the experimentally determined distributions indicated that this force response was at least very nearly normally distributed.

The distributions of four of the nine records of force normal to the direction of excitation were checked. Three of these records were for the high level narrow band and bandpassed cases in which rotation was substantial as evidenced by approximately equivalent normal and parallel force spectra (Figure 9). Of these three cases, the distributions for the two narrow band cases showed some fairly significant deviations from normality. A smoothed approximation to the trends of the distributions of normal force when significant rotation is present is shown in Figure 11. Figure 11 is essentially normal probability paper on which the cumulative distribution of a normally distributed variate plots as a straight line. The data for the two narrow band cases indicated a non-normal symmetrical distribution with lower "tails" and higher midpoint than for the normal density function. The normal force record for the high-level "Bandpassed" excitation case appeared normally distributed.

Though the evidence is limited, it appears that only when quite substantial, visually observable, rotation occurs, is there a substantial deviation of tank force response from normality.

3. Non-Stationary Random Excitation

A typical time-history of the non-stationary random excitation experiments is shown in Figure 12. It was apparent from the spectral analysis that if the bandpassed excitation used in the non-stationary case had been turned up and left on for a longer period of time, the amplitudes of force at 90° to the direction of excitation would have been about the same as those of force parallel to the excitation. Apparently, some time is required for the rotational instability to fully develop since the normal force amplitudes in Figure 12 are approximately 1/30 the parallel force amplitudes. At the beginning of the excitation, parallel force response, but little normal force response, is experienced. Parallel force response is abruptly increased by a violent displacement excursion and afterwards the normal force builds up and does not begin to decay until well after the excitation transient is over. For the most part, the normal and parallel forces are roughly in phase which indicates that the "burst" of excitation may have had the effect of shifting the node orientation of the first anti-symmetric mode. If shift in nodal position is the correct explanation for the normal forces in Figure 12, it is notable that the apparent phase relations between the two forces do not change during the decay; that is, the nodal position changed only in response to an external stimulus. The decay is in the first anti-symmetric mode.

Since relatively little has been done with non-stationary random processes (with a possible exception being Ref. 4) no further analysis of the present results was contemplated, or in fact, could be accomplished in the present program.

Perhaps the only important point is that this type of experiment may simulate actual fuel tank excitation in a booster rocket more nearly than has been done in the past, and despite the complexity of the excitation, nothing really unusual or unexpected on the basis of previous experience, was found to occur.

4. Cross-Spectral Analysis of Broadband Excitation Records

The primary objective in carrying out cross-spectral analyses on the broadband excitation data was to enable estimates to be made of the transfer function between tank displacement and parallel force (Eqn. 3). The analyses were carried out on all three "broadband" excitation records in accordance with the procedures outlined in the section on data reduction, and estimates were made of the transfer functions for each case (Ref. 4, 5).

Figure 13 shows the "input" and "output" spectra for the highest level of excitation. The "output" force spectrum has been "re-colored" but not corrected for instrumentation filter roll-off. The filter on the parallel force did not roll off appreciably until a frequency of 8 or 10. The roll-off of the force spectrum shown is as much due to the filter as to the roll-off or "input" (tank displacement). The force filter amplitude response is quite "flat" to a frequency of about 8 or 9 and has a phase response linear with frequency in this range. These characteristics facilitated correction of the derived transfer functions to the one desired; that is, the relation between tank displacement and unfiltered force.

The transfer functions derived from the cross-spectral analyses are shown in Figure 14 and compared with the undamped theory and with some sinusoidal excitation data from Ref. 3. Computed coherencies between parallel force and tank displacement are shown in Figure 15. The coherency is an index of linear dependence and the adequacy of the analysis. The notable feature of the coherencies of Figure 15 is the consistent "hole" near the first anti-symmetric mode. If the analysis is adequate the coherency levels of 0.1 displayed indicate that there is little or no linear dependence between tank displacement and parallel force up to a frequency of 2.5. This feature is independent of tank rms displacement level. The sample in which swirl instability seemed negligible ($\sigma = 5.3 \times 10^{-4}$), has quite respectable coherencies from $\Omega = 2.5$ upward and a definite drop below this frequency—in exactly the same manner as in the case where excitation energy was 100 times greater. The coherencies for the medium excitation level ($\sigma = 1.66\times10^{-3}$) are too low above a frequency of 3. The writer strongly suspects that the data reduction procedure is at fault in some way in this case, but the matter could not be pursued.

The correspondence of the derived transfer functions (Fig. 14) with theoretical above a frequency of 2.5 is good, well within 90% confidence bounds corresponding to the computed coherencies in each case, assuming, consistent with past experience, that in real tanks the peaks associated with $n = 1$ and higher modes nearly disappear. The derived transfer functions for frequencies above 7 from the three cases check one another and continue the ($\propto \Omega^2$) trend expected.

If it were not for the poor coherencies below a frequency of 2.5, the derived transfer functions might be categorized as "reasonable". As matters stand the fact that the amplitude response is in the correct cycle of the log scale near first anti-symmetric mode resonance may be purely a matter of chance. The trial analysis without the special pre-whitening filter resulted in coherencies around 0.1 at the first mode. If this was due wholely to poor resolution in the analysis, the pre-whitening process should have improved coherencies near first mode from "terrible" to at least "bad". In fact, the pre-whitening process made no significant improvement. In the author's opinion the coherencies presented in Figure 15 are likely to be of the correct magnitude. The lowest level broad-band excitation in the test program produced very little free surface activity of any sort. It was the initial hope that the extent of the validity of the linear hypthesis for first mode sloshing in an unbaffled tank could be established as a function of excitation level. This has not been realized in the present experiments. In fact, the evidence indicates that the linear hypothesis for first mode sloshing in an unbaffled tank may be invalid under any visible level of random excitation.

CONCLUDING REMARKS

In the present exploratory program the question of the possibility of "swirl" instability in an unbaffled circular cylindrical tank under random excitation has been answered in the affirmative and some evidence shown that large, visually observable, swirl might be predicted on the basis of a "threshold" excitation spectral density.

Attempts were made in the program to demonstrate the validity of the linear hypothesis for force response of an unbaffled tank under sufficiently low-level random lateral excitation. The fragments of data produced indicate, however, that if the hypothesis is reasonable at all for first mode sloshing, it would apply for almost vanishing excitation levels.

Some qualitative experiments on non-stationary random excitation were attempted in conjunction with the study of "swirl" instability under random excitation with no really unexpected results observed.

ACKNOWLEDGEMENTS

The author acknowledges with thanks the substantial contributions toward completion of this phase of the program made by Dr. H. Norman Abramson, Director, Department of Mechanical Sciences, in direction of efforts, by Messrs. J. E. Modisette and L. M. Yeakley, without whose expertise in instrumentation and the design of off-beat shaker systems nothing quantitative could have been accomplished, as well as Mr. D. C. Scheidt, whose care and patience with the actual experiments and analog data reduction cannot be overstated.

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APPENDIX

NOTES ON:

EXPLORATORY STUDIES OF LIQUID BEHAVIOR IN RANDOMLY EXCITED TANKS: LATERAL EXCITATION

Motion Picture Report No. 2
Contract No. NAS8-20319
Control No. DCN 1-6-57-01042(1F)

TYPE FILM: Silent - Color, 16 mm

PROJECTION TIME: Approximately 15 minutes at 16 frames/sec.

GENERAL:

The first sequences in the film show a general view of the tank and mechanical apparatus with a somewhat smaller field than that of Figure 1 of this report. Subsequent sequences showing the tank in motion were framed so that as large as possible a view of the free surface was obtained. The field of view is somewhat smaller than that of Figure 2 of this report. The camera angle is approximately 45° to the direction of excitation. For the comments which follow, rotation or lack of rotation was judged on the basis of fluid motion at the tank side adjacent to the near force balance shown in the films. A line connecting the near force balance with the one on the far side (visible through the tank in the film) is at 90° to the direction of excitation, or parallel to the expected position of the node for the first anti-symmetric mode.

All run sequences were taken at 32 frames per second so that the time scale, when the film is projected at 16 frames, is appropriate for an 30.7 inch (77 cm) tank in a 1 g acceleration field.

COMMENTS ON THE FILM SEQUENCES:

a. After identification titles and setup view, two sequences are shown of "Narrow Band Random Excitation". The excitation spectra for both are shown in a chart following the initial title. For both cases the excitation energy is concentrated in a narrow frequency band centered on the frequency of the first anti-symmetric mode. The difference between the two samples is in excitation level, the first, (Sample 20) is at maximum attainable level, the second (Sample 2) is at roughly 1/10 this <u>amplitude</u> level, or roughly 1/100 the maximum <u>energy</u> level.

The sequence following for Sample 20, under this specialized form of random excitation, shows large amplitude "swirl" qualitatively identical to that observed under sinusoidal excitation. A second anti-symmetric mode response may be observed superimposed upon the large amplitude first anti-symmetric response. The buildup and subsequent decay of violent motion which is characteristic of random phenomena may be noticed in the sequence as can the reversal of direction of rotation, characteristically observed under sinusoidal excitation.

The immediately following sequence, (Sample 21, 1/100 excitation energy) in contrast shows very little rotation, and in fact, relatively small response in the first anti-symmetric mode.

b. The next two sequences are titled and shown in a similar fashion. These involve "Band Passed Random" excitation at two levels. The difference from the preceding case is that approximately the same input energy is "spread out" over a frequency band 16 times as great as that preceding. The result (Sample 24) is almost no large rotational tendencies visible though substantial anti-symmetric mode response may be seen. The following sequence (Sample 25), for 1/100 the energy level of Sample 24, shows very small free surface activity of any sort.

c. The next pair of sequences involve "Broadband" random excitation. As may be observed from the chart, a relatively constant excitation spectral density was achieved over a frequency range including the first 100 free surface modal frequencies. The film sequence for maximum excitation level (Sample 33) shows a rather confused free surface rich in many higher modes with an occasional first anti-symmetric buildup, but no significant rotation. The corresponding low level excitation case (Sample 35) shows very little free surface activity.

d. The last section of the film involves samples of a qualitative study of non-stationary random excitation. This type of excitation was produced by varying the gain of the "bandpassed" excitation signal of Sample 24 from zero to maximum and back to zero. A typical oscillograph record of displacement and the resulting forces on the tank is shown. (It should be noted that the scale for force at 90° to displacement direction is about 1/10 that for force in the direction of displacement.)

Two examples of this case follow the chart. In both cases the free surface is calm at the start and each sequence is ended when the motion of the free surface has substantially decayed. Unfortunately, a slight camera vibration makes visual determination of the end of the <u>excitation</u> transient difficult to see. In both examples shown in the film, higher mode content is visible at the start, as well as the first anti-symmetric mode which persists for some time. No substantial rotation is observed.

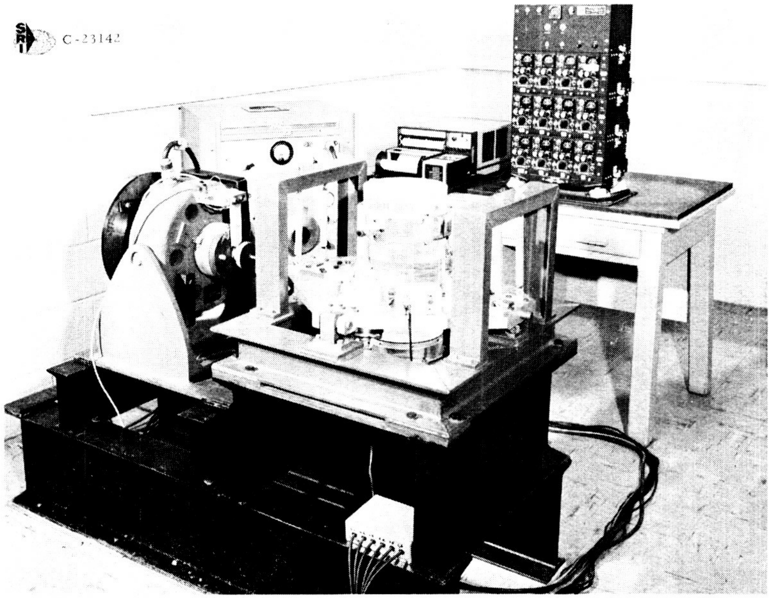

Figure 1. Experimental Apparatus For Lateral Sloshing, Ref. 3

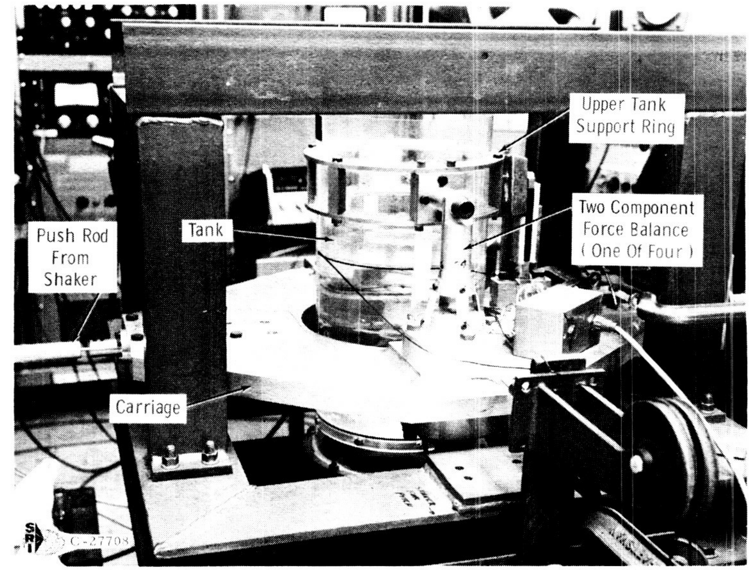

Figure 2. Tank, Force Balances And Supporting Apparatus

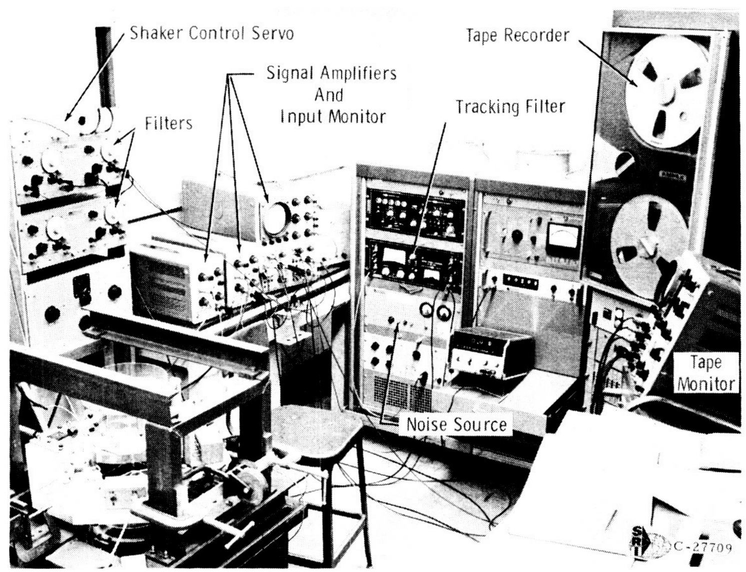

Figure 3. Experimental Setup

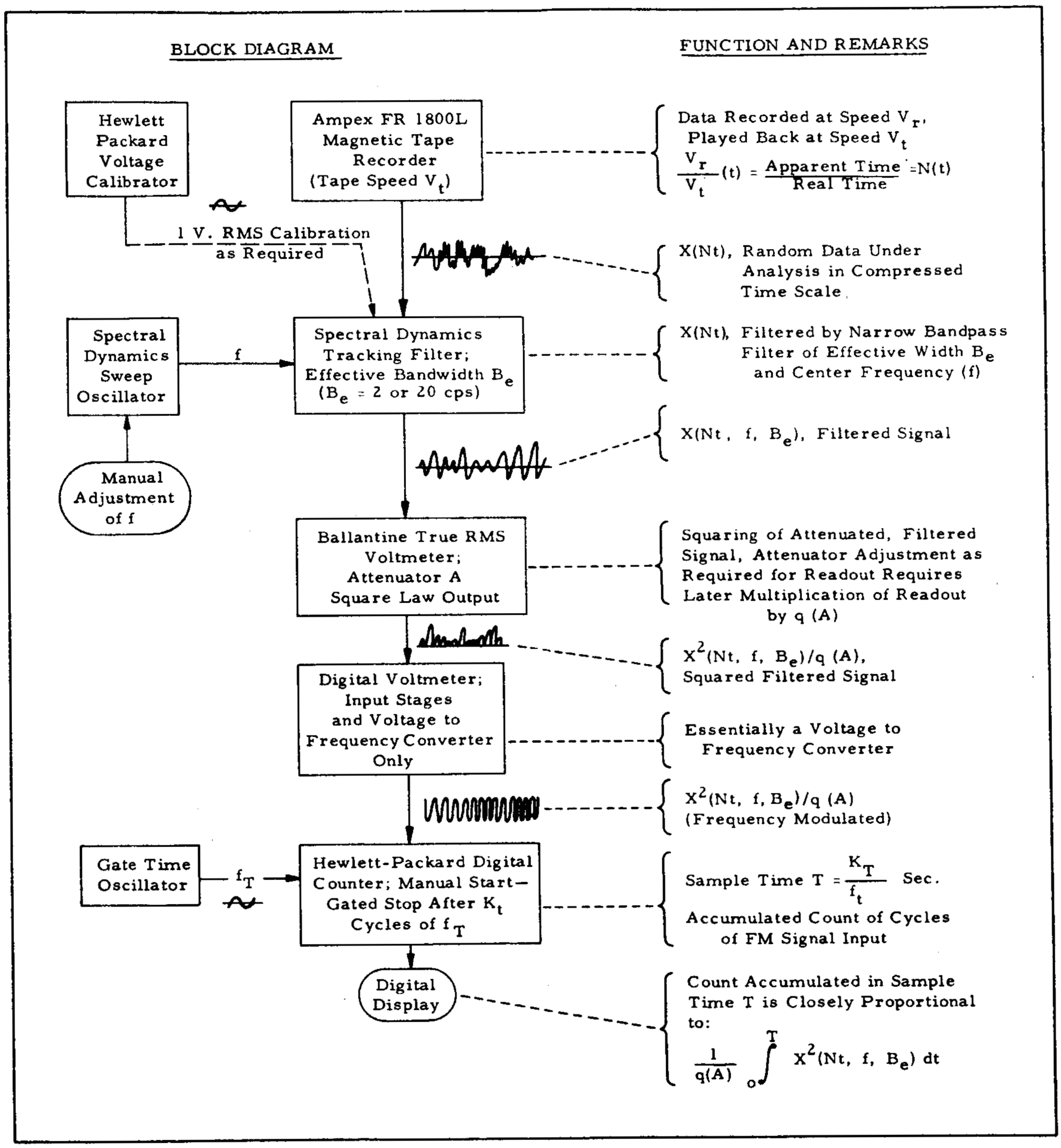

Figure 4. Spectrum Analyzer Block Diagram

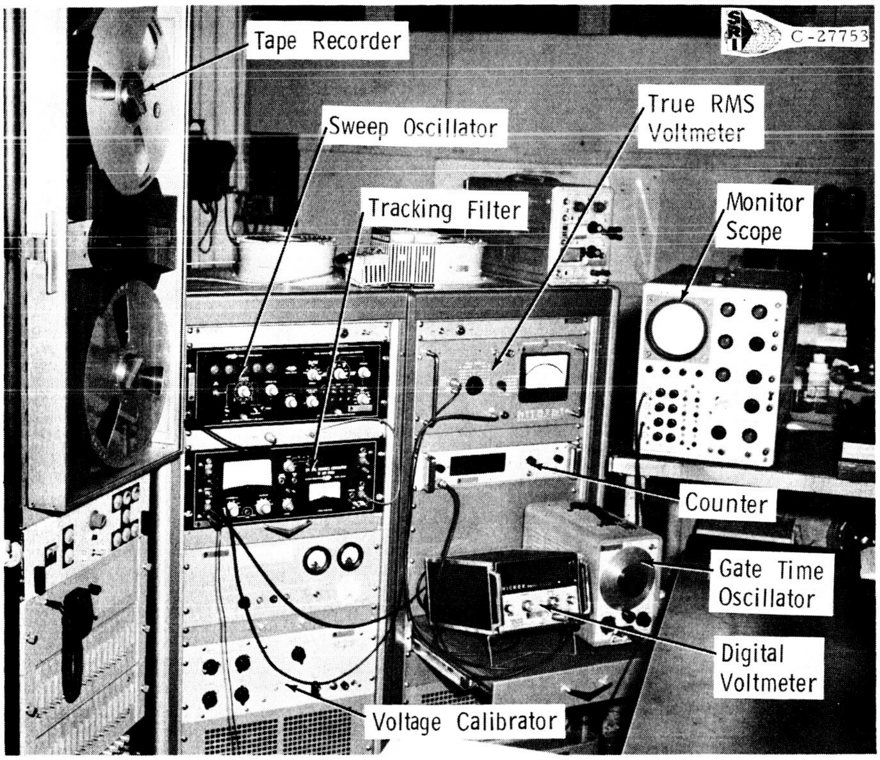

Figure 5. Spectrum Analysis Equipment

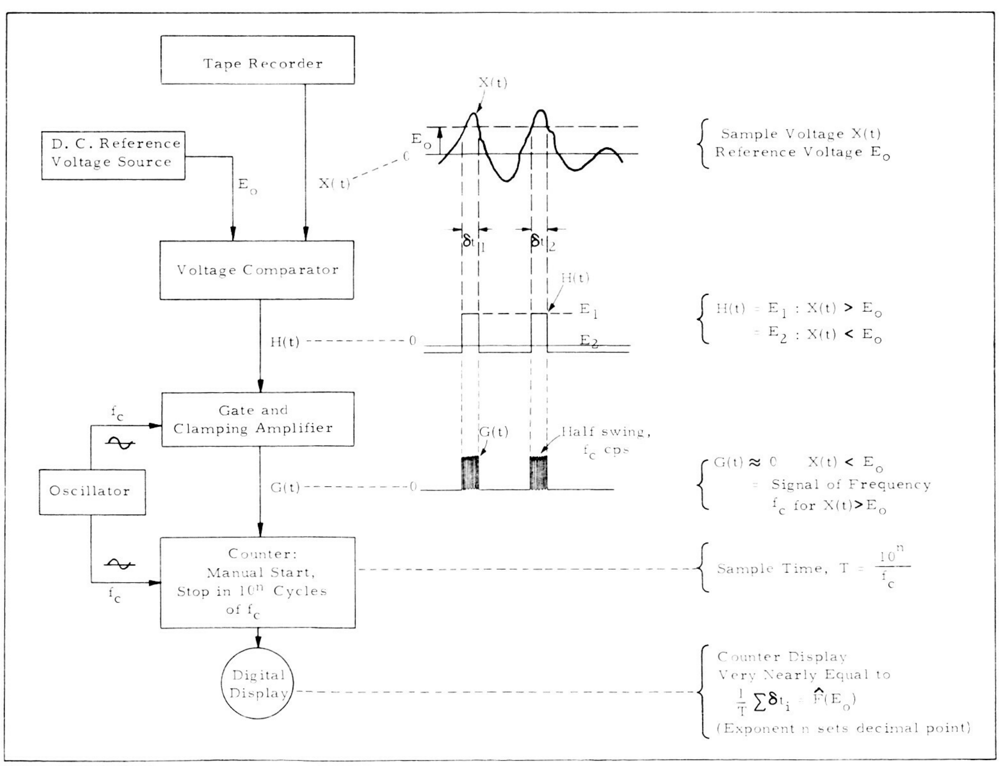

Figure 6. Block Diagram - Probability Distribution Estimation

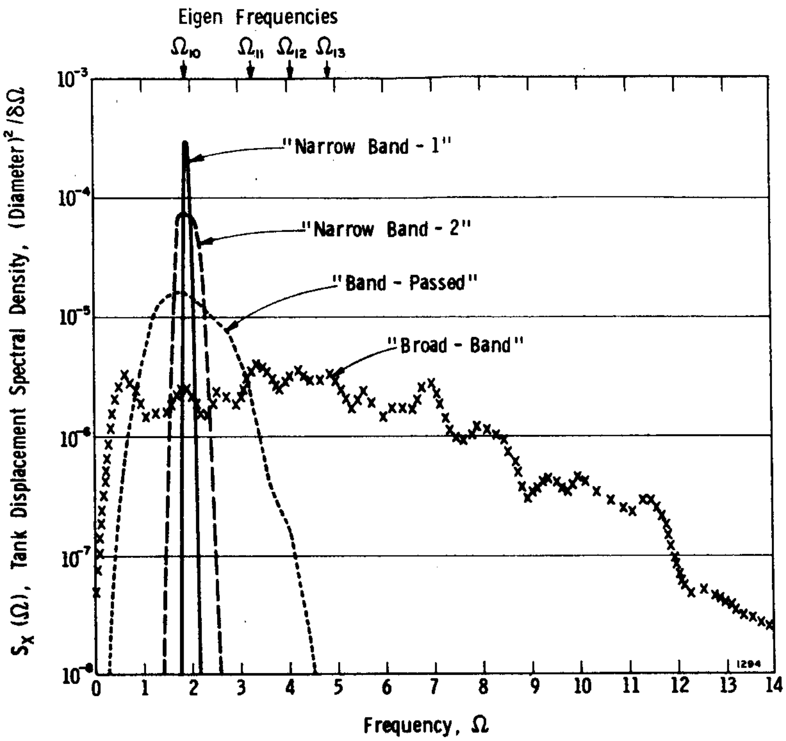

Figure 7. Excitation Spectra: Tank Displacement, Maximum Excitation Level

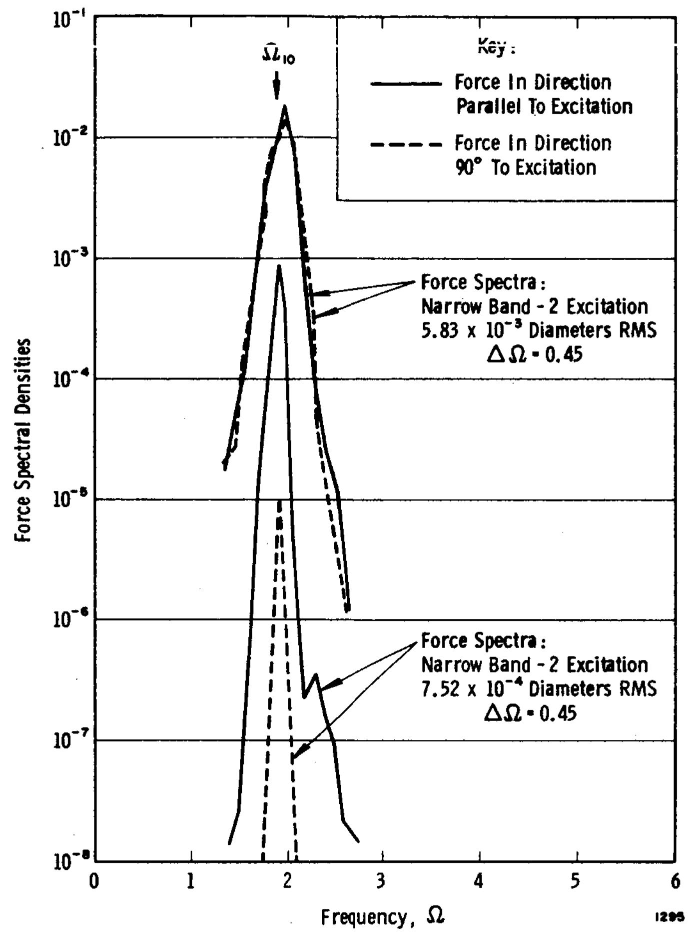

Figure 8. Force Spectra: Narrow Band - 2 Excitation

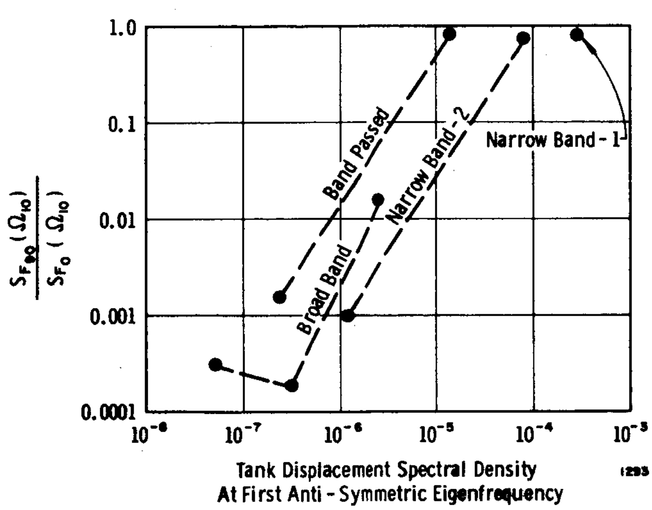

Figure 9. Ratio Of Force Spectral Density Peaks, (Normal And Parallel To Excitation) vs Tank Displacement Spectral Density At The First Anti - Symmetric Eigenfrequency

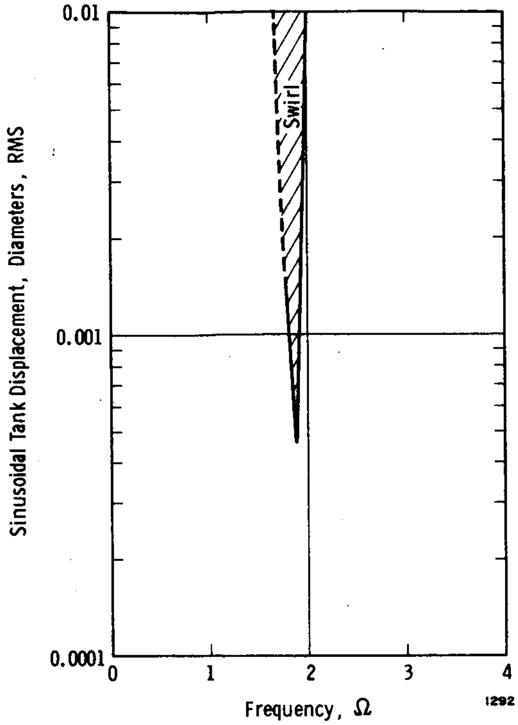

Figure 10. Swirl Stability Boundary From Ref. 3

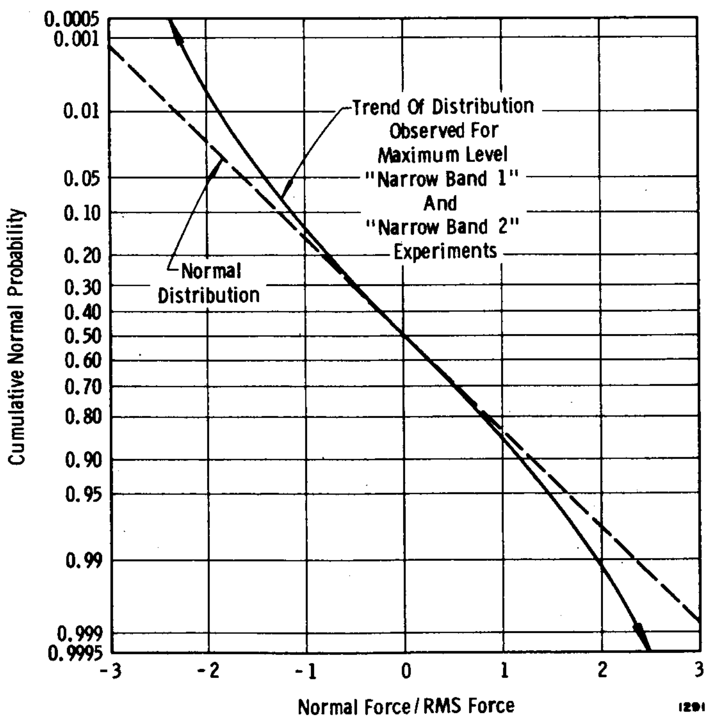

Figure 11. Probability Distribution Trends

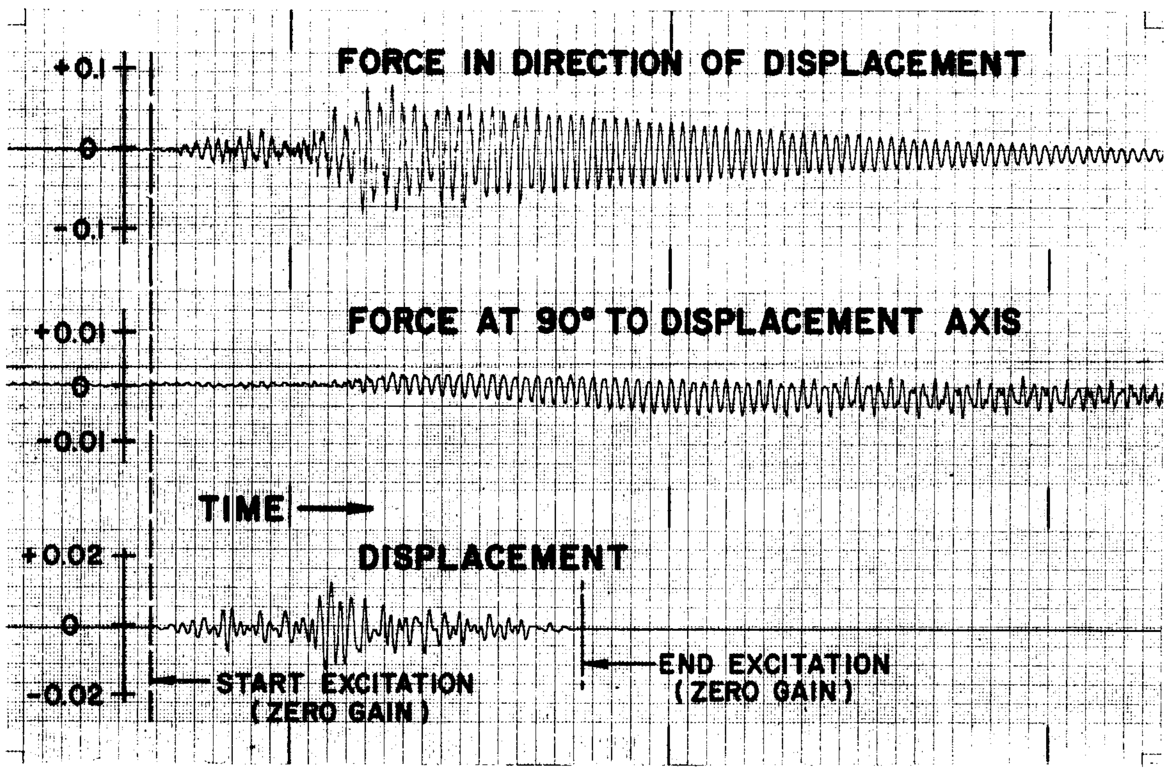

Figure 12. Typical Time History, Non-Stationary Random Excitation

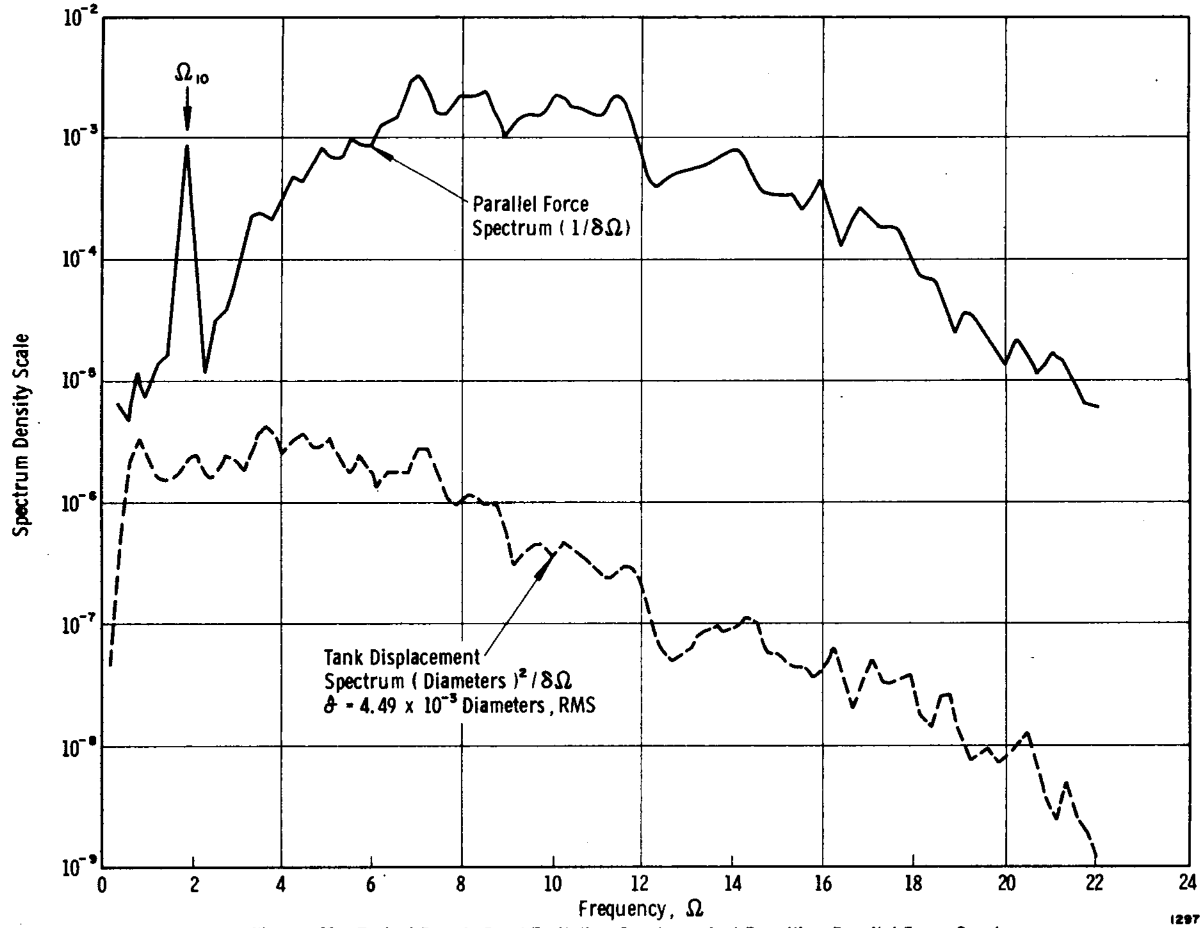

Figure 13. Typical Broad-Band Excitation Spectrum And Resulting Parallel Force Spectrum

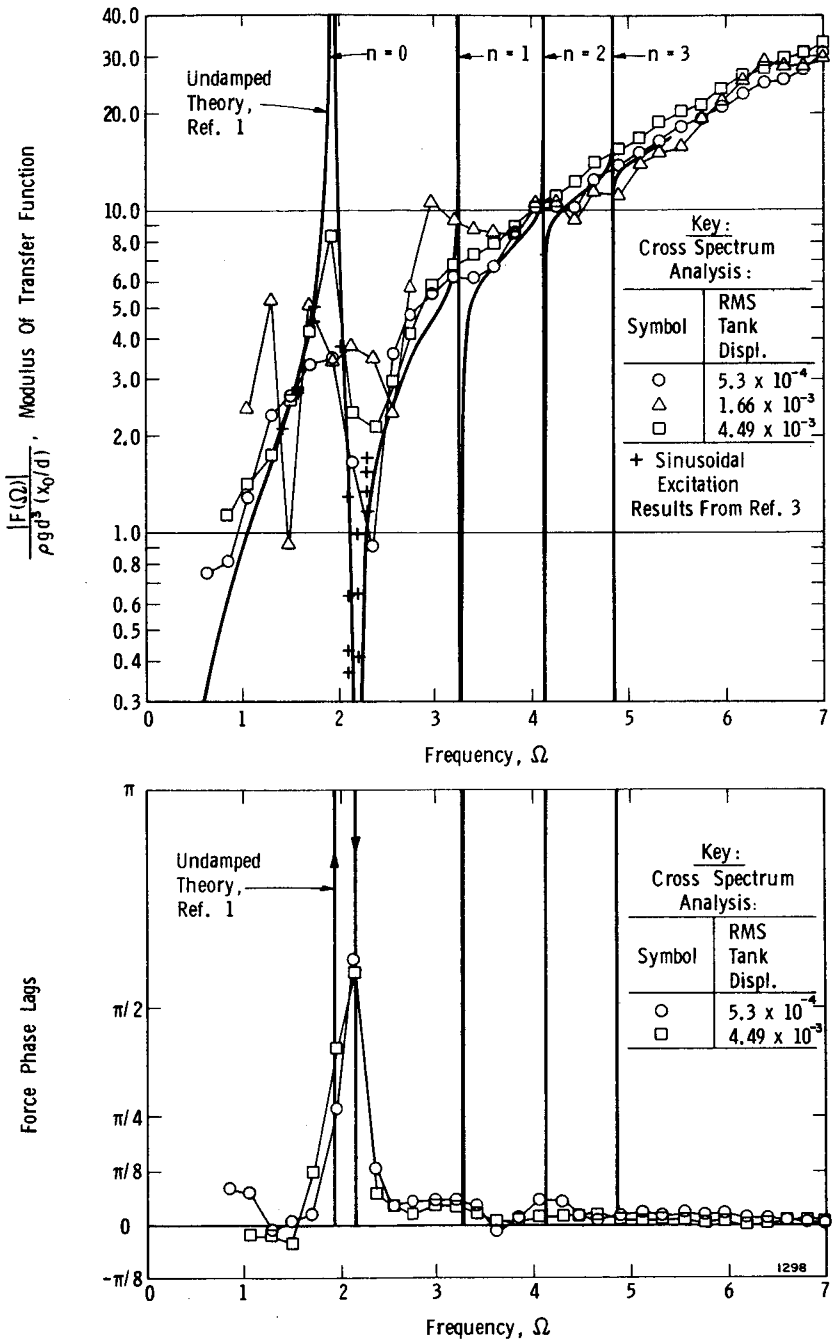

Figure 14. Tank Displacement - Force Transfer Functions Derived From Cross - Spectral Analysis

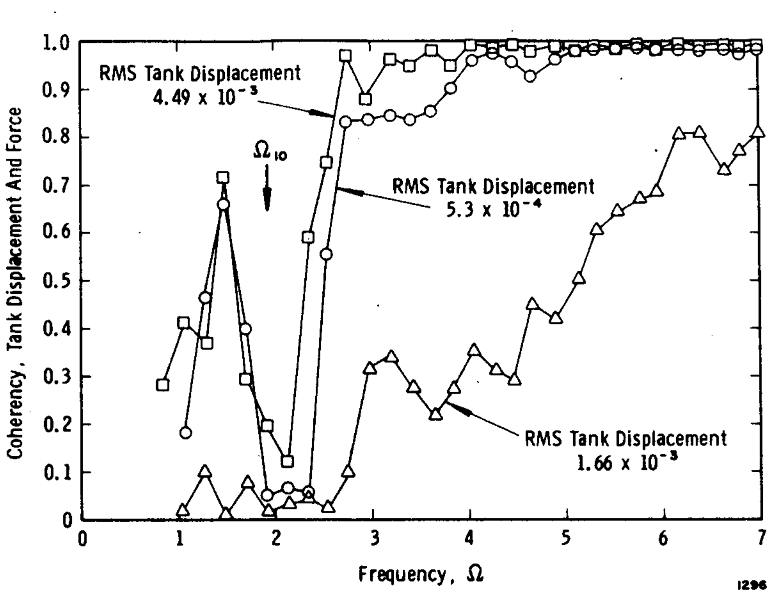

Figure 15. Estimated Tank Displacement-Force Coherencies
(Broad Band Excitation)